

BATU-EXAM

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at MET Bhujbal Knowledge City

Engg Maths 2 Department

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1) Solve $(D^6 - D^4)y = x^2$
 → Given:- $(D^6 - D^4)y = x^2$

A.E $m^6 - m^4 = 0$

$$m^4(m^2 - 1) = 0$$

$$m^4(m+1)(m-1) = 0$$

$m = 0, 0, 0, 0, 1, -1$

$$y_c = (c_1 + c_2x + c_3x^2 + c_4x^3)e^0x + (c_5e^{1x} + c_6e^{-1x})$$

$$y_c = c_1 + c_2x + c_3x^2 + c_4x^3 + c_5e^x + c_6e^{-x}$$

To find $y_p = \frac{1}{D^6 - D^4} \cdot x^2$

$$y_p = \frac{1}{D^4(D^2 - 1)} x^2$$

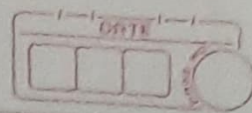
$$y_p = \frac{1}{D^4(1 - D^2)} x^2$$

Using $\frac{1}{1-x} = 1+x+x^2+x^3+x^4+\dots+x^n$
 here $x = D^2$

$$y_p = \frac{1}{D^4} \left(\frac{1}{1-D^2} \right) x^2$$

$$y_p = \frac{1}{D^4} [1 + D^2 + D^4 + D^6 + \dots] x^2$$

← multiply



$$y_p = -\frac{1}{D^4} [x^2 + 2 + 0 + 0]$$

$$y_p = -\frac{1}{D^3} \left[\frac{x^3}{3} + 2x + C \right]$$

$$= -\frac{1}{D^2} \left[\frac{x^4}{12} + \frac{2x^2}{2} + C_1x + C_2 \right]$$

$$y_p = -\frac{1}{D} \left[\frac{x^5}{60} + \frac{x^3}{3} + C_1x^2 + C_2x + C_3 \right]$$

$$y_p = -1 \left[\frac{x^6}{360} + \frac{x^4}{12} + C_1x^3 + C_2x^2 + C_3x + C_4 \right]$$

$$\therefore y = y_c + y_p$$

$$y = C_1 + C_2x + C_3x^2 + C_4x^3 + C_5e^x + C_6e^{-x}$$

$$\text{Solution} \rightarrow \left[\frac{x^6}{360} + \frac{x^4}{12} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4 \right]$$

2) Solve $(D^2 - 2D + 1)y = x \cdot e^x \cos x$

\rightarrow Given: $(D^2 - 2D + 1)y = x \cdot e^x \cos x$

A.E is $m^2 - 2m + 1 = 0$ $e^{ax} \cdot (v)$

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$\therefore y_c = (C_1 + C_2x)e^{ax}$$

~~Particular integral = $\frac{1}{(D-1)^2} [e^x (x \cos x)]$~~

$$\begin{aligned}
 &= e^x \cdot \frac{1}{(D+1)^2} (x \cdot \cos x) \\
 &= e^x \cdot \frac{1}{D^2} x \cdot \cos x \\
 &= e^x
 \end{aligned}$$

To find P.I = $\frac{1}{\phi(D)} F(x)$

$$= \frac{1}{D^2 - 2D + 1} x e^x \cos x$$

$$= e^x \frac{1}{D^2 - 2D + 1} x \cos x$$

put $D = D+1$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} x \cos x$$

$$= e^x \frac{1}{D^2 + 2D + 1 - 2D - 2 + 1} x \cos x$$

$$= e^x \frac{1}{D^2} x \cos x$$

$$= e^x \frac{1}{D} [x \sin x + \cos x]$$

$$= e^x [x \cos x + 2 \sin x]$$

$$= e^x (2 \sin x - \cos x)$$

$$y = CF + PI$$

$$= (C_1 + C_2 x) e^{2x} + e^{2x} (2 \sin x - \cos x)$$

3) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

→ Given: $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

i.e., $(D^2 + 1)y = \operatorname{cosec} x$

A.E is $m^2 + 1 = 0$

$m = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$

Compare with $y_c = c_1 y_1 + c_2 y_2$

$y_1 = \cos x, y_2 = \sin x$
 $y_1' = -\sin x, y_2' = \cos x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= (\cos x)(\cos x) - (\sin x)(-\sin x)$$

$$= \cos^2 x + \sin^2 x$$

$W = 1$

$$u = \int \frac{-y_2 \cdot \operatorname{cosec} x}{W} dx$$

$$= \int -\sin x \cdot \operatorname{cosec} x dx$$

$$= \int -\sin x \cdot \frac{1}{\sin x} dx$$

$$u = \int -1 dx$$

$$\boxed{u = -x}$$

$$\boxed{-u = x}$$

$$\begin{aligned}
 v &= \int y_1 \frac{x}{w} dx \\
 &= \int \cos x \cdot \frac{\cos x}{1} dx \\
 &= \int \cos^2 x \cdot \frac{1}{\sin x} dx \\
 &= \int \cot x dx
 \end{aligned}$$

$$v = \log \sin x$$

$$y_p = u y_1 + v y_2$$

$$= -x(\cos x) + \log \sin x (\sin x)$$

$$y_p = x \cos x + \sin x \cdot \log \sin x$$

$$y_p = -x \cos x + \sin x \cdot \log x - \sin x$$

$$y = y_c + y_p$$

$$= c_1 \cos x + c_2 \sin x + [-x \cos x + \sin x \cdot \log \sin x]$$

$$y = c_1 \cos x + c_2 \sin x - x \cos x + \sin x \log \sin x$$

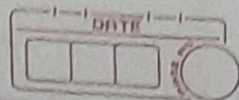
Q) Solve: $(D^2 - 2D + 1)y = x \cdot e^x \cdot \sin x$
 → Given: $(D^2 - 2D + 1)y = x e^{2x} \cdot \sin x$

$$A.E = D^2 - 2D + 1 = 0$$

$$(D-1)^2 = 0 \quad D = 1, 1,$$

or

$$m = 1, 1.$$



$$C.F. = (c_1 x + c_2) e^x$$

$$P.I. = \frac{1}{(D-1)^2} (x \cdot e^x \sin x)$$

$$= e^x \cdot \frac{1}{(D+1-1)^2} (x \sin x)$$

$$= e^x \frac{1}{D^2} (x \sin x)$$

$$= e^x \left[\frac{x-2b}{D^2} \right] \frac{1}{D} (\sin x)$$

$$= e^x \left[\frac{x-2}{D} \right] (-\sin x) = -e^x \left[\frac{x \sin x - 2 \sin x}{D} \right]$$

$$= -e^x [x \sin x + 2 \cos x]$$

Hence, the complete solution is

$$y = C.F. + P.I.$$

$$y = (c_1 x + c_2) e^x - e^x [x \sin x + 2 \cos x]$$

11) Solve $(D^2 + 2D + 1)y = e^{-x} \log x$ by method of variation of parameters.

→ Given:-

$$(D^2 + 2D + 1)y = e^{-x} \log x$$

$$A.E. \quad m^2 + 2m + 1 = 0$$

$$m = -1, -1 \dots$$

$$y_c = (c_1 + c_2 x) e^{-x}$$

$$y_c = c_1 e^{-x} + c_2 x e^{-x}$$

Compare with $yc = u y_1 + c_2 y_2$

$$y_1 = e^{-x}$$

$$y_2 = x \cdot e^{-x}$$

$$y_1' = -e^{-x}$$

$$y_2' = x \cdot \frac{d}{dx}(e^{-x}) + e^{-x} \frac{d}{dx}(x)$$

$$y_2' = -x \cdot e^{-x} + e^{-x}(1)$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x} & x \cdot e^{-x} \\ -e^{-x} & -x \cdot e^{-x} + e^{-x} \end{vmatrix}$$

$$W = -e^{-x} \cdot x e^{-x} + e^{-x} \cdot e^{-x} + e^{-x} \cdot x e^{-x}$$

$$W = e^{-x} \cdot e^{-x}$$

$$u = \int -y_2 \frac{x}{W} dx$$

$$= \int -x \cdot \frac{e^{-x} \cdot e^{-x}}{e^{-x} \cdot e^{-x}} \log x dx$$

$$u = -\int x \log x dx$$

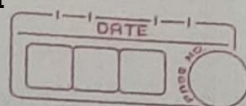
Using $\int u \cdot v dx = u \int v dx - \int u' v dx$

$$u = - \left[\log x \cdot \frac{x^2}{2} - \left[\frac{1}{x} \int \frac{x^2}{2} dx \right] \right]$$

$$u = - \left[\frac{x^2 \log x}{2} - \int \frac{x}{2} dx \right]$$

$$u = - \left[\frac{x^2 \log x}{2} - \frac{1}{2} \int x \right]$$

$$u = - \left[\frac{x^2 \log x}{2} - \frac{1}{2} \frac{x^2}{2} \right]$$



$$v = \int y_1 \frac{x}{w} dx$$

$$v = \int \frac{e^{-x} \cdot e^{-x} \log x}{e^{-x} \cdot e^{-x}} dx$$

$$v = \int \log x dx$$

$$v = x \log x - x$$

by VPM

$$y_p = u y_1 + v y_2$$

$$y_p = \left[- \left(\frac{x^2 \log x}{2} - \frac{1}{2} \frac{x^2}{2} \right) \right] e^{-x} + x \log x - x \left[x \cdot e^{-x} \right]$$

$$y = y_c + y_p$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + \left[- \left(\frac{x^2 \log x}{2} - \frac{1}{2} \frac{x^2}{2} \right) \right] e^{-x} + x \log x - x \left[x \cdot e^{-x} \right]$$

9) Solve : $\frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} + 9y = 5^x - \log 2$

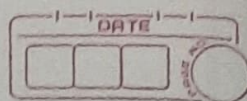
→ Step I :- The given diff eqⁿ :

$$F(D)y = F(x)$$

$$\text{i.e., } (D^2 + 6D + 9)y = 5^x - \log 2$$

The general solⁿ of the given diff eqⁿ is,

$$y = C.F + P.I = y_c + y_p \quad \text{--- (1)}$$



Step II: For CF, A.E. is:

$$p^2 + 6p + 9 = 0$$

$$(p+3)^2 = 0$$

$$p = -3, -3 \quad \text{or} \quad m = -3, -3.$$

$$CF = (C_1 + C_2x) e^{mix}$$

$$y_c = (C_1 + C_2x) e^{-3x} \quad \text{--- (2)}$$

Step III: For P.F:

$$P.I = \frac{1}{F(D)} \cdot f(x)$$

$$y_p = \frac{1}{p^2 + 6p + 9} (5^x - \log 2)$$

$$= \frac{1}{(p+3)^2} 5^x - \frac{1}{(p+3)^2} (\log 2) e^{0x}$$

$$= \frac{1}{(p+3)^2} e^{(\log 5)x} - \frac{1}{(p+3)^2} (\log 2) e^{0x}$$

use $(p = \log 5)$ $p = a = 0$

$$y_p = \frac{5^x}{(\log 5 + 3)^2} - \frac{\log 2}{9}$$

Step IV: From eq (2) & (3) the complete solution Eq (1) is,

$$y = (C_1 + C_2x) e^{-3x} + \frac{5^x}{(\log 5 + 3)^2} + \frac{\log 2}{9}$$

10) Solve $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 5y = 25x^2$.

→ Given: $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 5y = 25x^2$

$$F(D) y = F(x) \text{ it}$$

i.e., $(D^2 - 2D + 5)y = 25x^2$
 The general solⁿ of the given diff eqⁿ is

$$y = C.F. + P.I. = y_c + y_p \quad \text{--- (1)}$$

Step I: For C.F.

A.E is $D^2 + 2$

$$D^2 - 2D + 5 = 0$$

$$m^2 - 2D + 5 = 0$$

$$m = 1 \pm 2i$$

$$C.F. = e^{ax} [C_1 \cos \beta x + C_2 \sin \beta x]$$

$$\therefore y_c = e^x [C_1 \cos 2x + C_2 \sin 2x] \quad \text{--- (2)}$$

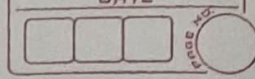
Step III: For P.I.,

$$P.I. = \frac{1}{F(D)} \cdot f(x)$$

$$y_p = \frac{1}{D^2 - 2D + 5} (25x^2)$$

$$= \frac{1}{5 - 2D + D^2} (25x^2)$$

(Take lowest degree term common).



$$y_p = \frac{1}{5} \left[1 + \left(\frac{-2D + D^2}{5} \right) \right]^{-1} (25x^2)$$

Expand the bracket by using $(1+z)^{-1}$

$$= 1 - z + z^2 - + \dots$$

Here, $z = \left(\frac{-2D + D^2}{5} \right)$

$$= \frac{1}{5} \left[1 - \left[\frac{-2D + D^2}{5} \right] + \left[\frac{-2D + D^2}{5} \right]^2 \right] (25x^2)$$

$$\left(\frac{4D^2 - 4D^3 + D^4}{25} \right)$$

The terms of degree 3 & 4 are neglected

$$y_p = \frac{1}{5} \left[1 + \frac{2D}{5} - \frac{D^2}{5} + \frac{(4D^2)}{25} \right] (25x^2)$$

$$= \frac{1}{5} \left[1 + \frac{2D}{5} - \frac{1}{25} D^2 \right] (25x^2)$$

25x ²
D 50x
D ² 50

$$= \frac{1}{5} \left[1(25x^2) + \frac{2D}{5} (25x^2) - \frac{1}{25} D^2 (25x^2) \right]$$

$$y_p = \frac{1}{5} \left[(25x^2) + \frac{2}{5} (50x) - \frac{1}{25} (50) \right]$$

$\underbrace{\hspace{10em}}_{(2 \times 10x)} \quad \underbrace{\hspace{2em}}_2$

$$y_p = \frac{1}{5} [25x^2 + 20x - 2]$$

$$= 5x^2 + 4x - \frac{2}{5} \quad \text{--- (3)}$$

Step IV: From eqⁿ (2) & (3), the complete solⁿ (1) is

$$y = e^x [C_1 \cos 2x + C_2 \sin 2x] + 5x^2 + 4x - \frac{2}{5}$$

Q 4) Solve : $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$

→ $x = e^z \rightarrow z = \log x$

$$x \frac{dy}{dx} = Dy ; \quad x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

where $D \equiv \frac{d}{dz}$

∴ given diff eqⁿ becomes

$$[D(D-1) - 3D + 5]y = e^{2z} \sin z$$

$$(D^2 - 4D + 5)y = e^{2z} \sin z$$

Which is L.D.E with constant coefficient
 $F(D)y = F(z)$

The g.s. of the given diff eqⁿ is
 $y = CF + PI \equiv y_c + y_p$ --- (1)

For CF: A.E. is $F(D) = 0$
 $D^2 - 4D + 5 = 0$

$$D = 2 \pm i \quad \text{or} \quad m = 2 \pm i$$

$$\therefore CF = e^{az} [C_1 \cos \beta z + C_2 \sin \beta z]$$

$$y_c = e^{2z} [C_1 \cos z + C_2 \sin z] \quad \text{--- (2)}$$

For P.I. $PI = \frac{1}{F(D)} \cdot f(z)$

$$y_p = \frac{1}{D^2 - 4D + 5} \cdot e^{2z} \sin z$$

Take e^{2z} outside & put $D = D+2$

$$y_p = e^{2z} \cdot \frac{1}{(D+2)^2 - 4(D+2) + 5} \sin z$$

$$(D^2 + 4D + 4 - 4D - 8 + 5)$$

$$y_p = e^{2z} \cdot \frac{1}{D^2 + 1} \sin z$$

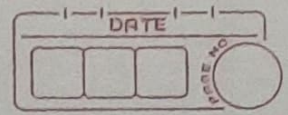
$$p^2 = -a^2 = -1$$

$$y_p = e^{2z} \cdot \frac{z}{2D} \sin z \quad [\because F(-D) = 0]$$

$$y_p = e^{2z} \cdot \frac{z}{2} \int \sin z \, dz = \frac{z \cdot e^{2z}}{2} (-\cos z)$$

$$\int \frac{1}{D} F(x) = \int F(x) dx$$

$$y_p = \frac{-ze^{2z} \cos z}{2} \quad \text{--- (3)}$$



From (2) & (3) the g.s. (1) is

$$y = e^{2z} [C_1 \cos z + C_2 \sin z] - \frac{z \cdot e^{2z} \cos z}{2}$$

Since, $z = \log x$; $e^z = x$

Ans. $y = x^2 [C_1 \cos(\log x) + C_2 \sin(\log x)] - \frac{x^2 \log x}{2}$
 $-\frac{x^2 \log x \cos(\log x)}{2}$

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