BATU-EXAM

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at MET Bhujbal Knowledege City

Engg Maths 2 Department

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Tutorial No. 3

1) Solve $(D^6 - D^4)y = x^2$

→ Given :- $(D^6 - D^4)y = x^2$

$$A.E \quad m^6 - m^4 = 0$$

$$m^4(m^2 - 1) = 0$$

$$m^4(m+1)(m-1) = 0$$

$$m = 0, 0, 0, 0, 1, -1$$

$$y_c = (c_1 + c_2x + c_3x^2 + c_4x^3)e^0x + (c_5e^{1x} + c_6x^{-1x})$$

$$y_c = c_1 + c_2x + c_3x^2 + c_4x^3 + c_5e^x + c_6e^{-x}$$

To find $y_p = \frac{1}{D^6 - D^4} \cdot x^2$

$$y_p = \frac{1}{D^4(D^2-1)x^2}$$

$$y_p = \frac{-1}{D^4(1-D^2)} \cdot x^2$$

Using $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots x^n$

Here $x = D^2$

$$y_p = \frac{-1}{D^4}\left(\frac{1}{1-D^2}\right)x^2$$

$\frac{1}{x}$

multiply

$$y_p = \frac{-1}{D^4}\left[1 + D^2 + D^4 + D^6 + \dots\right]x^2$$

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$$y_p = -\frac{1}{D^4}\left[x^2 + 2 + 0 + 0\right]$$

$$y_p = -\frac{1}{D^3}\left[\frac{x^3}{3} + 2x + c\right]$$

$$= -\frac{1}{D^2}\left[\frac{x^4}{12} + \frac{\cancel{2}x^2}{\cancel{2}} + c_1 x + c_2\right]$$

$$y_p = -\frac{1}{D}\left[\frac{x^5}{60} + \frac{x^3}{3} + \frac{c_1 x^2}{2} + c_2 x + c_3\right]$$

$$y_p = -1\left[\frac{x^6}{360} + \frac{x^4}{12} + \frac{c_1 x^3}{6} + \frac{c_2 x^2}{2} + c_3 x + c_4\right]$$

$$\therefore y = y_c + y_p$$

$$y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 e^{x} + c_6 e^{-x}$$

Solution $\rightarrow \left[\frac{x^6}{360} + \frac{x^4}{12} + \frac{c_1 x^3}{6} + \frac{c_2 x^2}{2} + c_3 x + c_4\right]$

2) Solve $(D^2 - 2D + 1)y = x \cdot e^{x} \cos x$

$\rightarrow$ Given:- $(D^2 - 2D + 1)y = x \cdot e^{x} \cos x$

A.E is $m^2 - 2m + 1 = 0$ $\qquad e^{ax} \cdot (v)$

$(m-1)^2 = 0$

$m = 1, 1$

$\therefore y_c = (c_1 + c_2 x)e^{x}$

~~Particular integral~~ $\cancel{= \frac{1}{(D-1)^2} \cdot \left[e^{x}(x\cos x)\right]}$

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$$\cancel{= e^x \cdot \frac{1}{(D+1-1)^2} (x \cdot \cos x)}$$

$$\cancel{= e^x \cdot \frac{1}{D^2} x \cdot \cos x}$$

$$\cancel{= e^x}$$

To find $P.I = \frac{1}{\phi(D)} \cdot F(x)$

$$= \frac{1}{D^2 - 2D + 1} \cdot x e^x \cos x$$

$$= e^x \cdot \frac{1}{D^2 - 2D + 1} x \cos x$$

put $D = D + 1$

$$= e^x \cdot \frac{1}{(D+1)^2 - 2D + D + 1} x \cos x$$

$$= e^x \cdot \frac{1}{D^2 + \cancel{2D} + 1 - \cancel{2D} + 1 - 2} x \cos x$$

$$= e^x \cdot \frac{1}{D^2} x \cos x$$

$$= e^x \cdot \frac{1}{D} [x \sin x + \cos x]$$

$$= e^x [-\cos x + 2 \sin x]$$

$$= e^x (2 \sin x - \cos x]$$

$$y = CF + PI$$

$$= (c_1 + c_2 x) e^x + e^x (2 \sin x - \cos x)$$

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3) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

→ Given :- $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

i.e., $(D^2+1)y = \operatorname{cosec} x$

A.E is $m^2 + 1 = 0$

$m = \pm i$

$y_c = \cancel{c_1e} \; c_1 \cos x + c_2 \sin x$

Compare with $y_c = c_1y_1 + c_2y_2$

$y_1 = \cos x, \quad y_2 = \sin x$
$y_1' = -\sin x, \quad y_2' = \cos x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= (\cos x)(\cos x) - (\sin x)(-\sin x)$
$= \cos^2 x + \sin^2 x$

$\boxed{W = 1}$

$u = \int -\frac{y_2 X}{W} dx$

$= \int -\sin x \cdot \frac{\operatorname{cosec} x}{1} dx$

$= \int -\sin x \cdot \frac{1}{\sin x} dx$

$u = \int -1 dx \qquad \boxed{u = -x}$

$\boxed{\cancel{-u = -x}}$

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$$v = \int y_1 \frac{X}{W} dx$$

$$= \int \cos x \cdot \frac{\operatorname{cosec} x}{1} dx$$

$$= \int \cos x \frac{1}{\sin x} dx$$

$$= \int \cot x \, dx$$

$$v = \log \sin x$$

$$y_p = u y_1 + v y_2$$

$$= -x(\cos x) + \log \sin x (\sin x)$$

$$y_p = x \cos x + \sin x \cdot \log \sin x$$

$$y_p = -x \cos x + \sin x \cdot \log x \sin x$$

$$y = y_c + y_p$$

$$= c_1 \cos x + c_2 \sin x + [-x \cos x + \sin x \cdot \log \sin x]$$

$$\boxed{y = c_1 \cos x + c_2 \sin x - x \cos x + \sin x \log \sin x}$$

6) Solve: $(D^2 - 2D + 1) y = x \cdot e^x \cdot \sin x$

→ Given:- $(D^2 - 2D + 1) y = x e^x \cdot \sin x$

$$A.E = D^2 - 2D + 1 = 0$$

$$(D-1)^2 = 0 \qquad D = 1, 1$$

or

$$m = 1, 1.$$

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$$CF = (c_1x + c_2)e^x$$

$$P.I = \frac{1}{(D-1)^2}(x.e^x \sin x)$$

$$= e^x \cdot \frac{1}{(D+1-1)^2}(x\sin x)$$

$$= e^x \frac{1}{D^2}(x\sin x)$$

$$= e^x\left[x - \frac{2D}{D^2}\right]\frac{1}{D^2}(\sin x)$$

$$= e^x\left[x - \frac{2}{D}\right](-\sin x) = -e^x\left[x\sin x - \frac{2}{D}\sin x\right]$$

$$= -e^x[x\sin x + 2\cos x]$$

Hence, the complete solution is

$$y = C_F + P_I$$

$$y = (c_1x + c_2)e^x - e^x[x\sin x + 2\cos x]$$

11) Solve $(D^2+2D+1)y = e^{-x}\log x$ by method of variation of parameters.

→ Given :-

$$(D^2+2D+1)\,y = e^{-x}\log x$$

$$A.E \quad m^2+2D+1=0$$

$$m=-1,-1$$

$$y_c = (c_1 + c_2x)e^{-x}$$

$$y_c = c_1e^{-x} + c_2x.e^{-x}$$

x. logsin x]

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Compare with $y_c = u y_1 + c_2 y_2$

$$y_1 = e^{-x} \qquad y_2 = x \cdot e^{-x}$$

$$y_1' = -e^{-x} \qquad y_2' = x \cdot \frac{d}{dx}(e^{-x}) + e^{-x} \frac{d}{dx}(x)$$

$$y_2' = -x \cdot e^{-x} + e^{-x}(1)$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x} & x \cdot e^{-x} \\ -e^{-x} & -x \cdot e^{-x} + e^{-x} \end{vmatrix}$$

$$W = -e^{-x} \cdot x e^{-x} + e^{-x} \cdot e^{-x} + e^{-x} \cdot x e^{-x}$$

$$W = e^{-x} \cdot e^{-x}$$

$$u = \int \frac{-y_2 X}{W} dx$$

$$= \int - \frac{x \cdot \cancel{e^{-x} \cdot e^{-x}}}{\cancel{e^{-x} \cdot e^{-x}}} \log x \, dx$$

$$u = -\int x \log x \, dx$$

Using $\int u \cdot v \, dx = u \int v - \int d \, u \int v$

$$u = -\left[\log x \cdot \frac{x^2}{2} - \left[\frac{1}{x} \int \frac{x^2}{2} dx \right] \right]$$

$$u = -\left[\frac{x^2}{2} \log x - \int \frac{x}{2} dx \right]$$

$$u = -\left[\frac{x^2}{2} \log x - \frac{1}{2} \int x \right]$$

$$u = -\left[\frac{x^2}{2} \log x - \frac{1}{2} \frac{x^2}{2} \right]$$

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$$v = \int y_1 \frac{X}{W}\, dx$$

$$v = \int \frac{e^{-x} \cancel{e^{-x}} \log x}{\cancel{e^{-x}} \cdot e^{-x}}\, dx$$

$$v = \int \log x\, dx$$

$$v = x\log x - x$$

by VPM

$$y_p = u y_1 + v y_2$$

$$y_p = \left[-\left(\frac{x^2}{2}\log x - \frac{1}{2}\frac{x^2}{2}\right)\right] e^{-x} + x\log x - x\left[x \cdot e^{-x}\right]$$

$$y = y_c + y_p$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + \left[-\left(\frac{x^2}{2}\log x - \frac{1}{2}\frac{x^2}{2}\right)\right] e^{-x}$$
$$+ x\log x - x\left[x \cdot e^{-x}\right].$$

9) Solve : $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5^x - \log 2$

→ Step I :- The given diff eq^n :

$$F(D)\, y = F(x)$$

i.e., $(D^2 + 6D + 9)\, y = 5^x - \log 2$

The general sol^n of the given diff eq^n is,

$$y = C.F + P.I = y_c + y_p \quad \text{——} \quad (1)$$

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Step II: For CF, A.E is:

$$D^2+6D+9=0$$
$$(D+3)^2=0$$

$$D=-3,-3 \quad \text{or} \quad m=-3,-3.$$

$$CF=(C_1+C_2x)e^{m_1x}$$
$$y_c=(C_1+C_2x)e^{-3x} \quad \text{——— (2)}$$

Step III: For P.I:

$$P.I=\frac{1}{f(D)}\cdot f(x)$$

$$y_p=\frac{1}{D^2+6D+9}(5^x-\log 2)$$
$$=\frac{1}{(D+3)^2}5^x-\frac{1}{(D+3)^2}(\log 2)e^{0x}$$

$$=\frac{1}{(D+3)^2}e^{(\log 5)x}-\frac{1}{(D+3)^2}(\log 2)e^{0x}$$

use $(D=\log 5)$ $\quad D=a=0$

$$y_p=\frac{5^x}{(\log 5+3)^2}-\frac{\log 2}{9}$$

Step IV: From eq (2) & (3) the complete solution Eq^n (1) is,

$$y=(C_1+C_2x)e^{-3x}+\frac{5^x}{(\log 5+3)^2}+\frac{\log 2}{9}$$

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10) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 25x^2$.

→ given :- $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 25x^2$

$$F(D)y = F(x)$$

i.e., $(D^2 - 2D + 5)y = 25x^2$

The general solⁿ of the given diff eqⁿ is

$$y = CF + P.I = y_c + y_p \quad \text{--- (1)}$$

Step I :- For C.F.

A.E is ~~$D^2 + 2$~~ $D^2 - 2D + 5 = 0$

$$m^2 - 2D + 5 = 0$$

$$m = 1 \pm 2i$$

$$Cf = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$$

$$\therefore y_c = e^{x}[c_1 \cos 2x + c_2 \sin 2x] \quad \text{--- (2)}$$

Step II : For P.I,

$$PI = \frac{1}{F(D)} \cdot f(x)$$

$$y_p = \frac{1}{D^2 - 2D + 5}(25x^2)$$

$$= \frac{1}{5 - 2D + D^2}(25x^2)$$

(Take lowest degree term common).

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$$= \frac{1}{5\left[1+\left(\frac{-2D+D^2}{5}\right)\right]}(25x^2)$$

$$y_p = \frac{1}{5}\left[1+\left(\frac{-2D+D^2}{5}\right)\right]^{-1}(25x^2)$$

Expand the bracket by using $(1+z)^{-1}$

$$= 1 - z + z^2 - + \dots$$

Here, $z = \left(\frac{-2D+D^2}{5}\right)$

$$= \frac{1}{5}\left[1 - \left[\frac{-2D+D^2}{5}\right] + \underbrace{\left[\frac{-2D+D^2}{5}\right]^2}_{\left(\frac{4D^2 - 4D^3 + D^4}{25}\right)}\right](25x^2)$$

The terms of degree 3 & 4 are neglected

$$y_p = \frac{1}{5}\left[1 + \frac{2}{5}D \underbrace{- \frac{D^2}{5} + \frac{1}{25}(4D^2)}_{\frac{5}{5\times5}D^2}\right](25x^2)$$

$$= \frac{1}{5}\left[1 + \frac{2}{5}D - \frac{1}{25}D^2\right](25x^2)$$

1	$25x^2$
D	$50x$
D^2	50

$$= \frac{1}{5}\left[1(25x^2) + \frac{2}{5}D(25x^2) - \frac{1}{25}D^2(25x^2)\right]$$

$$y_p = \frac{1}{5}\left[(25x^2) + \underbrace{\frac{2}{5}(50x)}_{(2\times 10x)} - \underbrace{\frac{1}{25}(50)}_{2}\right]$$

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$$y_p = \frac{1}{5}\left[25x^2 + 20x - 2\right]$$

$$= 5x^2 + 4x - \frac{2}{5} \longrightarrow (3)$$

Step IV : From eq^n (2) & (3) the complete sol^n (1) is

$$y = e^{x}\left[c_1 \cos 2x + c_2 \sin 2x\right] + 5x^2 + 4x - \frac{2}{5}$$

Q 4) Solve : $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$

$\rightarrow$ $x = e^z \longrightarrow z = \log x$

$$x\frac{dy}{dx} = Dy \quad ; \quad x^2\frac{d^2y}{dx^2} = D(D-1)y$$

where $D \equiv \frac{d}{dz}$

$\therefore$ given diff eq^n becomes

$$[D(D-1) - 3D + 5]y = e^{2z}\sin z$$

$$(D^2 - 4D + 5)y = e^{2z}\sin z$$

Which is L.D.E with constant coefficient $f(D)y = f(z)$]

The g.s. of the given diff eq^n is

$$y = CF + PI \equiv y_c + y_p \text{ ——— } (1)$$

For CF : AE is $F(D) = 0$

$$D^2 - 4D + 5 = 0$$

$$D = 2 \pm i \quad \text{or} \quad m = 2 \pm i$$

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$$\therefore CF = e^{\alpha z}[c_1 \cos \beta z + c_2 \sin \beta z]$$

$$y_c = e^{2z}[c_1 \cos z + c_2 \sin z] \quad \text{—— (2)}$$

For P.I. $\quad PI = \frac{1}{F(D)} \cdot F(z)$

$$y_p = \underbrace{\frac{1}{D^2 - 4D + 5}}\cdot e^{2z} \sin z$$

Take e^{2z} outside & put $D = D+2$

$$y_p = e^{2z} \cdot \underbrace{\frac{1}{(D+2)^2 - 4(D+2) + 5}}_{(D^2+4D+4-4D-8+5)} \sin z$$

$$y_p = e^{2z} \cdot \underbrace{\frac{1}{D^2+1}}_{D^2 = -a^2 = -1} \sin z$$

$$y_p = e^{2z} \frac{z}{2D} \sin z \qquad [\because F(-1) = 0]$$

$$y_p = e^{2z} \cdot \frac{z}{2} \int \sin z \, dz = \frac{z \cdot e^{2z}}{2} (-\cos z)$$

$$\text{——} \left[\because \frac{1}{D} F(x) = \int F(x) dx \right.$$

$$y_p = \frac{-z e^{2z}}{2} \cos z \quad \text{—— (3)}$$

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From ② & ③ the g.s. ① is

$$y = e^{2z}[c_1 \cos z + c_2 \sin z] - z \cdot \frac{e^{2z}}{2} \cos z$$

Since, $z = \log x$; $e^{z} = x$

Ans. $$y = x^2[c_1 \cos(\log x) + c_2 \sin(\log x)] \cancel{-\frac{x^2}{2}\log} - \frac{x^2}{2} \log x \cos(\log x)$$

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